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PRESSURE-RECOVERY CURVE IN THE FILTRATION OF A GAS–CONDENSATE MIXTURE

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The dynamics of fluids for condensed gas deposits is described by equations of two-phase multicomponent filtration with phase transitions that do not admit analytical solutions. This complicates the interpretation of pressure-recovery curves (PRC) for production wells, since to process experimental data one should have an explicit theoretical formula with one or several free parameters that characterize the reservoir properties.

Let us consider the case where the pressure is assumed to be constant at the hole bottom, and dynamical processes in a well can be ignored. We assume that a stationary flow of a two-phase gas–condensate mixture (described in [1–3]) is realized prior to the shut-down of a well. Upon shutting down, a complicated process of pressure and substance redistribution occurs in a porous medium, which terminates when the mixture pressure and composition become equal to the values of these parameters at the injection contour. Reservoir engineering experience shows that the pressure equalization proceeds much faster than that of composition equalization. Thus, the time it takes for the condensate accumulated in the porous medium near the well to resolve is several orders of magnitude greater than the time of observation of pressure rise. This makes it possible to simulate the pressure recovery within the framework of the linear theory of perturbations, since the nonlinear character of the filtration equations of a gas-condensate mixture is mainly associated with phase transitions.

Applicability of the linear theory is the key assumption of the present study. It allows one to derive an explicit asymptotic formula for PRC.

We consider an $(M+1)$-component mixture $(M>0)$. Let n_i be the corresponding molar densities of the components. Hereafter the subscripts i, j, and k take on the values $0,\ldots,M$, and the subscripts α and β take on the values $1,\ldots,M$. It is assumed everywhere that summation is performed over repeat subscripts. We study only isothermal processes and, therefore, the dependence of all mechanical and thermodynamic quantities on temperature is ignored.

For homogeneous states of the mixture, statistical physics [4] enables one to calculate the free energy per unit volume $f=f(n_i)$, which is a smooth and one-valued function of the components' density n_i. The thermodynamic relations

$$df=\varkappa_i dn_i, \qquad f=-p+\varkappa_i n_i, \tag{1}$$

are valid, where $\varkappa_i$ are the chemical potentials and p is the pressure. From (1) follows the Duhem equality

$$dp=n_i d\varkappa_i. \tag{2}$$

If the function $f=f(n_i)$ is convex, the homogeneous states of the mixture are then thermodynamically stable in volume. For a two-phase system of the gas–condensate type, this function is not convex. In this case, thermodynamic stability of the homogeneous state n_i is checked in considering virtual separations into the liquid and gas phases n_i and n_{il}:

$$n_i=sn_{il}+(1-s)\,n_{ig}, \tag{3}$$

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where s is the volume portion of the liquid phase (condensate). If the quantity

$$f^* = sf(n_{il}) + (1 - s)f(n_{ig}) \tag{4}$$

is less than $f(n_i)$, the homogeneous state is thermodynamically unstable. The heterogeneous state (3) that ensures a minimum value of (4) is stable.

Let us redetermine the free energy per unit volume $f = f(n_i)$ in a two-phase medium using formula (4), which specifies the free-energy value in a stable heterogeneous state. Thus, instead of the original function $f = f(n_i)$, we consider its convex shell (denoted below by the same symbol). The new free energy is a twice-differentiable function. It is easy to verify that with the new definition of free energy, thermodynamic relations (1) and (2) still hold, and now $\ae_i$ and p are the chemical potentials and the pressure in each phase of the two-phase medium. Thus, free energy allows one to calculate the molar phase densities n_{ig} and n_{il}, the volume portion of the condensate s, and also the chemical potentials and the pressure in the mixture from the known molar density n_i of the mixture.

Let an unsteady filtration flow of cylindrical symmetry occur in a uniform isotropic reservoir with porosity m and permeability k in the vicinity of a production well. Denote the distance to the well axis by r. The local conservation laws for the components hold [5]:

$$\partial_t(mn_i) + r^{-1}\partial_r(rJ_i) = 0. \tag{5}$$

With capillary forces ignored, according to Darcy's law, the following relations are valid for flows J_i:

$$J_i = -kK_i\partial_r p, \qquad K_i = f_g n_{ig}\mu_g^{-1} + f_l n_{il}\mu_l^{-1}. \tag{6}$$

The quantities n_i, n_{ig}, and n_{il} are related by relation (3). In relation (6), $\mu_g = \mu_g(n_{ig})$ and $\mu_l = \mu_l(n_{il})$ are the shear viscosities of the gas and of the condensate respectively, and $f_g = f_g(s)$ and $f_l = f_l(s)$ are the permeabilities of the gas and of the condensate. The quantities s, n_{ig}, and n_{il} are thermodynamic functions of the densities n_i. Therefore, system (5) is complete for unknown functions $n_i = n_i(t, r)$.

We shall consider the boundary conditions. Let r_w be the well radius over a drill bit and r_0 be the injection-contour radius. In view of the formulation of the problem, one should impose the following boundary conditions on the pressure:

$$p\Big|_{r=r_w} = p_w, \quad t \leqslant 0, \qquad \partial_r p\Big|_{r=r_w} = 0, \quad t > 0; \tag{7}$$

$$p\Big|_{r=r_0} = p_0, \qquad p_w < p_0. \tag{8}$$

The so-called retrograde condensation is characteristic of gas-condensate deposits: under pressure decay, the gas phase becomes thermodynamically unstable and a liquid condensate falls out [6]. The condensate acquires hydrodynamical mobility only near production wells where it can occupy a considerable part of the pore volume. Denote the concentrations of gas-phase components that are fed to the well injection contour by c_{i0} ($c_{i0} > 0$). If p_d is the pressure of the onset of condensation for a mixture of this composition, then, generally speaking, $p_0 \geqslant p_d$.

We impose the following boundary condition on the mixture composition $c_i = n_i / \sum\limits_{j=0}^{M} n_j$:

$$c_i\Big|_{r=r_0} = c_{i0}. \tag{9}$$

We introduce into consideration the quantities

$$n_g = \sum_{i=0}^{M} n_{ig}, \quad n_l = \sum_{i=0}^{M} n_{il}, \quad K = \sum_{i=0}^{M} K_i = f_g n_g \mu_g^{-1} + f_l n_l \mu_l^{-1}, \quad C_i = K_i / K.$$

By definition, the quantities C_i can be interpreted as the concentrations of the components of a mixture that separates under pressure p into the same phases as the original mixture. The corresponding complete molar density N, the partial densities N_i, and the liquid-phase portion S are determined by the formulas

$$N = K/(f_g\mu_g^{-1} + f_l\mu_l^{-1}), \ N_i = NC_i = (1 - S)n_{ig} + Sn_{il}, \ S = f_l\mu_l^{-1}/(f_g\mu_g^{-1} + f_l\mu_l^{-1}). \tag{10}$$

We consider an inverse problem of determination of the parameters of the initial mixture n_i from the known values of C_i and p. With known C_i and p, the thermodynamics makes it possible to find the values of n_{ig}, n_{il}, and S. To determine the value of s, one can use Eq. (10). For unique solvability of Eq. (10), in the class of processes considered (where the gas phase is necessarily present) we assume that there is no mobility threshold for the liquid phase. In the last case, the right-hand side of Eq. (10) is a strictly monotonic function of s, for $s < 1$. According to (3), one can calculate the partial densities for the entire mixture n_i from the known s, n_{ig}, and n_{il}.

We introduce the coordinate $\eta = \ln(r/r_w)$. Problem (5)–(9) can then be rewritten as follows:

$$m\partial_t(r^2 n_i) - k\partial_\eta(KC_i\partial_\eta p) = 0; \tag{11}$$

$$p\Big|_{\eta=0} = p_w, \quad t \leqslant 0, \qquad \partial_\eta p\Big|_{\eta=0} = 0, \quad t > 0; \tag{12}$$

$$p\Big|_{\eta=\zeta} = p_0; \tag{13}$$

$$C_i\Big|_{\eta=\zeta} = c_{i0}, \tag{14}$$

where $\zeta = \ln(r_0/r_w)$. Boundary condition (14) follows from (9) with allowance for the inequality $p_0 \geqslant p_d$.

Recall the properties of the steady-state solutions of problem (11)–(14) [1–3] that are derived if condition (12) is replaced by

$$p|_{\eta=0} = p_w. \tag{15}$$

In the stationary case, from (11) and (14) it follows that

$$C_i = c_{i0}, \qquad K\partial_\eta p = q = Q/(2\pi kh). \tag{16}$$

Here the integration constant Q is the well debit in moles per unit time and h is the reservoir capacity. According to (16), the pressure is determined from the ordinary differential equation $p = p(\eta + \alpha, q, c_{i0})$. The free parameters α and q are found from boundary conditions (13) and (15).

Thus, if boundary conditions (13) and (15) are not used, one can obtain a steady-state solution for densities in the functional form $n_i = n_i(\eta + \alpha, q, c_{i0})$.

Below, the values calculated for the stationary solution are asterisked. Determine the vector fields $e_i^j = e_i^j(\eta)$ (j is the field number and i is the component number) by the formulas

$$e_i^0(\eta) = \frac{\partial}{\partial\alpha} n_i(\eta + \alpha, q, c_{i0})\Big|_{\alpha=0} = \left(\frac{\partial n_i}{\partial \eta}\right)_*, \; e_i^\alpha(\eta) = \left(\frac{\partial n_i}{\partial C_\alpha}\right)_{p*} = m_i^\alpha - m_j^\alpha \left(\frac{\partial p}{\partial n_j}\right)_* (\partial_\eta p_*)^{-1} e_i^0; \tag{17}$$

$$m_i^\alpha(\eta) = \frac{\partial}{\partial c_{\alpha 0}} n_i(\eta, q, c_{i0}). \tag{18}$$

In differentiating with respect to the concentrations in formulas (17) and (18), one should take into account that C_α and $c_{\alpha 0}$ depend actually only on M in view of the normalized equalities

$$\sum_{i=0}^{M} C_i = \sum_{i=0}^{M} c_{i0} = 1.$$

It is obvious that the vector fields $e_i^j(\eta)$ form the basic set in the $(M+1)$-dimensional space for each η. Using this basic set, one can expand any other vector field. Let $\delta n_i = \delta n_i(t, \eta)$ be the small perturbations of the stationary solution. Let us expand with respect to the basic set $\delta n_i(t, \eta) = e_i^j(\eta) x_j(t, \eta)$ and substitute it into the dynamics equation for perturbations following from (11): $m\partial_t(r^2\delta n_i) - k\partial_\eta(\delta K C_i \partial_\eta p + K\delta C_i \partial_\eta p + KC_i\partial_\eta \delta p) = 0$.

We then obtain the system of $(M+1)$ linear equations for $(M+1)$ unknown functions $x_j(t, \eta)$ describing the perturbation dynamics:

$$m(kq)^{-1} r^2 G_\alpha^j \partial_t x_j - \partial_\eta x_\alpha = 0; \tag{19}$$

$$m(kq)^{-1}r^2\rho^j\partial_t x_j - \partial_\eta(\nu^\alpha x_\alpha) - \partial_\eta^2 x_0 = 0$$
$$\left[\rho^j = \sum_{i=0}^{M} e_i^j,\quad \nu^\alpha = \left(\frac{\partial \ln K}{\partial C_\alpha}\right)_{p*},\quad G_\alpha^j = e_\alpha^j - c_{i0}\rho^j\right]. \tag{20}$$

The pressure-field perturbations are representable by the formula

$$\delta p = \beta x_0, \quad \beta = \partial_\eta p_*. \tag{21}$$

The boundary and initial conditions for the functions $x_j(t,\eta)$ are obtained from (12)–(14) and (21) as

$$(\partial_\eta x_0 + \gamma x_0)\Big|_{\eta=0} = -1, \quad t > 0; \tag{22}$$

$$x_i\Big|_{\eta=\zeta} = 0; \tag{23}$$

$$x_i = 0, \quad t \leqslant 0. \tag{24}$$

In relations (22), $\gamma = \partial_\eta \ln\beta\Big|_{\eta=0}$. It is convenient to solve problem (19), (20), and (22)–(24) using the method of Fourier transform. We separate the variables:

$$x_0(t,\eta) = \int \exp(i\omega t) z(\omega,\eta)d\omega, \quad x_\alpha(t,\eta) = \int \exp(i\omega t) y_\alpha(\omega,\eta)d\omega.$$

Substituting these expressions into relations (19), (20), and (22)–(24), we obtain the following system of ordinary differential equations and boundary conditions:

$$i\omega Ay + i\omega v_1 z - \partial_\eta y = 0; \tag{25}$$

$$(i\omega h_1 + h_2)y + i\omega f_1 z - \partial_\eta^2 z = 0; \tag{26}$$

$$(\partial_\eta z + \gamma z)\Big|_{\eta=0} = i(2\pi)^{-1}(\omega - i\varepsilon)^{-1}; \tag{27}$$

$$z\Big|_{\eta=\zeta} = 0; \tag{28}$$

$$y\Big|_{\eta=\zeta} = 0. \tag{29}$$

Here y is a column vector with components y_α, A is an $M \times M$ matrix with components $A_{\alpha\beta} = m(kq)^{-1}r^2G_\alpha^\beta$, v_1 is a column vector with components $v_{1\alpha} = m(kq)^{-1}r^2G_\alpha^0$, h_1 is a row vector with components $h_{1\alpha} = (m(kq)^{-1}r^2\rho^\alpha - \nu^\beta A_{\beta\alpha})$, h_2 is a row vector with components $h_{2\alpha} = (-\partial_\eta\nu^\alpha)$, $f_1 = (m(kq)^{-1}r^2\rho^0 - \nu^\beta v_{1\beta})$, and ε is an infinitesimal positive quantity. It should be noted that the matrix A and the vector v_1 vanish identically when the pressure p exceeds the pressure of onset of condensation p_d.

We consider the incomplete problem (25) and (29). Define the matrix-valued function $U = U(\eta,\xi)$ as a solution of the Cauchy problem: $\partial_\eta U(\eta,\xi) = i\omega A(\eta)U(\eta,\xi)$ and $U(\xi,\xi) = 1$.

A solution of incomplete problem (25) and (29) is found by the formula

$$y(\omega,\eta) = i\omega\int_\eta^\zeta U(\eta,\xi)v_1(\xi)z(\omega,\xi)d\xi,$$

whose substitution into Eq. (26) yields the integrodifferential equation

$$i\omega(i\omega L_1 + L_2)z + i\omega f_1 z - \partial_\eta^2 z = 0, \tag{30}$$

where the operators L_a ($a = 1$ and 2) are determined by the formulas

$$(L_a z)(\eta) = \int_\eta^\zeta h_a(\eta)U(\eta,\xi)v_1(\xi)z(\omega,\xi)d\xi.$$

We thus obtained the complete problem (27), (28), and (30) to determine the pressure dynamics. We shall seek intermediate asymptotics of PRC for the case $\zeta = +\infty$ and replace condition (28) by the boundedness condition for the function z at infinity. We also assume that as $\eta \to +\infty$, the function $r^{-2}f_1(\eta)$ tends to a positive constant φ, while the function $f_0(\eta) = (f_1(\eta) - r^2\varphi)$ decreases rapidly. Define the function $\alpha = \alpha(\omega)$ from the relations $\alpha^2 = i\omega\varphi$ and $\operatorname{Re}\alpha \geqslant 0$ and let $R = R(\omega)$ be the next operator in $L^2(0, +\infty)$:

$$(Rg)(\eta) = \int\limits_0^{+\infty} G(\eta, \eta_1) g(\eta_1) d\eta_1, \quad g \in L^2(0, +\infty).$$

Here

$$G(\eta, \eta_1) = \begin{cases} D_1 K_0(\alpha r), & \eta \geqslant \eta_1, \\ D_2 K_0(\alpha r) + D_3 I_0(\alpha r), & \eta < \eta_1; \end{cases}$$

K_a and I_a are the MacDonald functions [7], and the coefficients $D_a = D_a(\eta_1)$ are found from the system of linear equations

$$G(\eta_1 + 0, \eta_1) - G(\eta_1 - 0, \eta_1) = 0, \quad (\partial_\eta G + \gamma G)\Big|_{\eta=0} = 0, \quad \partial_\eta G\Big|_{\eta=\eta_1+0} - \partial_\eta G\Big|_{\eta=\eta_1-0} = 1.$$

Then, from Eq. (30) and boundary conditions (27) and (28), we obtain the following operator equation for the function $z(\eta)$:

$$(1 + i\omega T)z = z_0, \tag{31}$$

where $T = T(\omega) = R(i\omega L_1 + L_2 + f_0)$ and $z_0(\omega, \eta) = i(2\pi)^{-1} K_0(\alpha r)/(\gamma K_0(\alpha r_w) - \alpha r_w K_1(\alpha r_w))$.

Recall that PRC are experimentally determined in the time range from tens of seconds to hours, which corresponds to the frequency range

$$10^{-4}\ \text{Hz} \leqslant |\omega| \leqslant 10^{-1}\ \text{Hz}. \tag{32}$$

Therefore, to obtain an asymptotic formula for PRC, it is sufficient to approximate the function z in frequency range (32), and then, according to (21), calculate the pressure recovery using the formula

$$\Delta p(t) = d_1 \int \exp(i\omega t)\psi(\omega) d\omega \qquad \left(\psi = z\Big|_{\eta=0}, \quad d_1 = \beta\Big|_{\eta=0}\right).$$

Let us consider Eq. (31). The operator $T(\omega)$ is interpreted as a linear continuous mapping onto the semi-axis $\eta \geqslant 0$ in the space of continuous bounded functions. Its norm can be estimated by numerical simulation of steady-state flows using the method of [2]. As a result, it appears that in the range of (32), we have $|\omega|\,\|T\| \ll 1$. Therefore, it is admissible to set $z \approx z_0$. In the expression for z_0, we retain only the main terms with low frequencies using the asymptotic functions for the MacDonald functions [7]:

$$\psi \approx z_0\Big|_{\eta=0} \approx i(2\pi)^{-1}(\omega - i\varepsilon)^{-1} \ln(2^{-1}\alpha r_w)(\gamma \ln(2^{-1}\alpha r_w) + 1)^{-1}.$$

The results of numerical simulation of steady-state flows show that the value of $\tau = r_w^2\varphi^{-1}$ is close, in order of magnitude, to 10 sec, while the dimensionless quantity γ in the case where the seam pressure p_w is much lower than the pressure of onset of condensation p_d is of the order of 10^{-2}. Therefore, in the range of (32), we have $|\gamma \ln(2^{-1}\alpha r_w)| \ll 1$. With allowance for the last remark, we obtain the expression for ψ whose analytical form coincides with that obtained in the problem of PRC under one-phase filtration [8]. By this analogy, one can write at once the asymptotic relation for PRC:

$$\Delta p(t) = 2^{-1} d_1 (\ln(t\tau^{-1}) + \mathrm{C}), \tag{33}$$

where C is the Euler constant. We rewrite the formula with separation of the debit dependence in explicit form:

$$\Delta p(t) = 2^{-1} q d_2 (\ln(t\tau^{-1}) + \mathrm{C}), \quad d_2 = K_*^{-1}\Big|_{\eta=0}. \tag{34}$$

Thus, in the case of gas–condensate mixture filtration, the PRC form is similar to the case of one-phase liquid filtration. However, unlike the one-phase case, to determine the permeability k using (33) or (34), it is necessary to perform an independent numerical simulation for steady-state flow (for example, to determine the quantities d_2 and τ).

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